

Expansion of massive scalar one-loop integrals to $\mathcal{O}(\varepsilon^2)$

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We report on the results of an ongoing calculation of massive scalar one-loop one-, two-, three- and four-point integrals up to $\mathcal{O}(\varepsilon^2)$ which are needed in the NNLO calculation of heavy hadron production.

1. Introduction

The full next-to-leading order (NLO) QCD corrections to hadroproduction of heavy flavors have been completed as early as 1988 [1,2]. They have raised the leading order (LO) estimates [3] but were still below the experimental results (see e.g. [4]). In a recent analysis theory moved closer to experiment [4]. A large uncertainty in the NLO calculation results from the freedom in the choice of the renormalization and factorization scales. The dependence on the factorization and renormalization scales is expected to be greatly reduced at next-to-next-to-leading order (NNLO). This will reduce the theoretical uncertainty. Furthermore, one may hope that there is yet better agreement between theory and experiment at NNLO.

2. General remarks

In Fig. 1 we show one generic diagram each for the four classes of gluon induced contributions that need to be calculated for the NNLO corrections to hadroproduction of heavy flavors. They involve the two-loop contribution (Fig. 1a), the loop-by-loop contribution (Fig. 1b), the one-loop gluon emission contribution (Fig. 1c) and, finally, the two gluon emission contribution (Fig. 1d). We mention that there is an interesting subclass of

the diagrams in Fig. 1c where the outgoing gluon is attached directly to the loop. One then has a five-point function which, when folded with the corresponding tree graph contribution, has to be calculated up to $\mathcal{O}(\varepsilon^2)$.

In our work we have concentrated on the loop-by-loop contributions exemplified by Fig. 1b. Specifically, working in the framework of dimensional regularization, we are in the process of calculating $\mathcal{O}(\varepsilon^2)$ results for all scalar massive one-loop one-, two-, three- and four-point integrals that are needed in the calculation of hadronic heavy flavour production. It should be clear from the flavour flow of the diagrams that, apart from the two mass scales set by the kinematics of the process there is only one explicit heavy mass scale in the problem. The integration is generally done by writing down the Feynman parameter representation for the corresponding integrals, integrating over Feynman parameters up to the last remaining integral, expanding the integrand of the last remaining parametric integral in terms of the dimensional parameter ε and doing the last parametric integration on the coefficients of the expansion. Because the one-loop integrals exhibit infrared (IR)/collinear (M) singularities up to $\mathcal{O}(\varepsilon^{-2})$ one needs to know the one-loop integrals up to $\mathcal{O}(\varepsilon^2)$ since the one-loop contributions appear in product form in the loop-by-loop contributions.

The aim of our project is thus to compute all one-loop contributions to the two processes

$$1. \quad g + g \rightarrow Q + \bar{Q}$$

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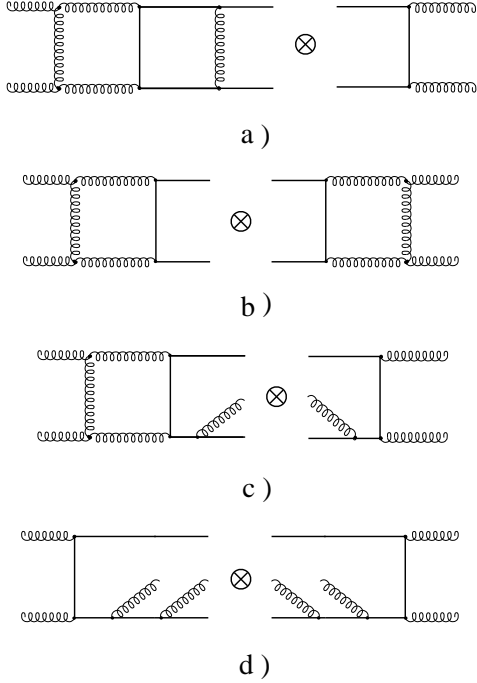


Figure 1. Exemplary gluon fusion diagrams for the NNLO calculation of heavy hadron production

2. $q + q \rightarrow Q + \bar{Q}$

up to $\mathcal{O}(\varepsilon^2)$.

Regarding the four classes of diagrams in Fig.1 one might then say in a very loose sense that we are aiming to calculate one-fourth of the NNLO partonic contributions to heavy hadron production. Nevertheless, calculating this one-fourth of the full problem already allows one to obtain a glimpse of the complexity that is waiting for us in the full NNLO calculation. This complexity does in fact reveal itself in terms of a very rich polylogarithmic structure of the Laurent series expansion of the scalar one-loop integrals as will be discussed later on.

In dimensional regularization there are three different sources that can contribute positive ε -powers to the Laurent series of the one-loop amplitudes. These are

1. Laurent series expansion of scalar one-loop integrals
2. evaluation of the spin algebra of the loop amplitudes bringing in the n -dimensional metric contraction $g_{\mu\nu}g^{\mu\nu} = n = 4 - 2\varepsilon$
3. Passarino-Veltman decomposition of tensor integrals involving again the metric contraction $g_{\mu\nu}g^{\mu\nu} = n = 4 - 2\varepsilon$

Concerning the first item the $\mathcal{O}(\varepsilon^2)$ calculation of the necessary one-, two- and three-point one-loop integrals for the loop-by-loop part of NNLO QCD calculation have now been completed by us. Two of the three massive one-loop four-point integrals have also been done leaving us with one remaining four-point integral which is presently being worked out. We hope to be able to present complete results on this part of the NNLO calculation in the near future [5].

Concerning the last two items (spin algebra and Passarino-Veltman decomposition) there exist some partial results on this part of the NNLO calculation which will be given at the end of this presentation.

Apart from the present discussion of NNLO contributions to heavy hadron production the calculation of massive scalar loop integrals up to a given positive power of ε is of interest also in other contexts. For example, if the one-loop integrals appear as subdiagrams in a given divergent Feynman diagram one again needs to avail of the positive ε -powers of the subdiagram. This is of relevance for the calculation of two-loop counter terms. Another example is the reduction of a given set of loop integrals to master integrals by the integration-by-parts technique [6]. In the reduction one may encounter explicit inverse powers of ε which implies that one has to evaluate the master integrals up to positive ε -powers.

3. Laurent series expansion of scalar one-loop integrals

In Table 1 we provide a list of the one-loop scalar one-, two-, three- and four-point integrals that need to be evaluated up to $\mathcal{O}(\varepsilon^2)$ in NNLO heavy hadron production. In column 2 the integrals are identified using the notation of [2].

Table 1

List of scalar n-point functions up to $\mathcal{O}(\varepsilon^2)$

	Nomenclature of Beenakker <i>et al.</i> [2]	Our nomenclature	Novelty	Comments
1-point	$A(m)$	A	–	Re \checkmark
2-point	$B(p_4 - p_2, 0, m)$	B_1	–	Re \checkmark
	$B(p_3 + p_4, m, m)$	B_2	–	Re, Im \checkmark
	$B(p_4, 0, m)$	B_3	–	Re \checkmark
	$B(p_2, m, m)$	B_4	–	Re \checkmark
	$B(p_3 + p_4, 0, 0)$	B_5	–	Re, Im \checkmark
3-point	$C(p_4, p_3, 0, m, 0)$	C_1	new	Re, Im \checkmark
	$C(p_4, -p_2, 0, m, m)$	C_2	new	Re \checkmark
	$C(-p_2, p_4, 0, 0, m)$	C_3	–	Re \checkmark
	$C(-p_2, -p_1, 0, 0, 0)$	C_4	–	Re, Im \checkmark
	$C(-p_2, -p_1, m, m, m)$	C_5	new	Re, Im \checkmark
	$C(p_3, p_4, m, 0, m)$	C_6	–	Re, Im \checkmark
4-point	$D(p_4, -p_2, -p_1, 0, m, m, m)$	D_1	new	Re, Im
	$D(-p_2, p_4, p_3, 0, 0, m, 0)$	D_2	new	Re, Im \checkmark
	$D(-p_2, p_4, -p_1, 0, 0, m, m)$	D_3	new	Re \checkmark

When writing down a Laurent series expansion of these integrals in terms of ε -powers one needs to introduce a short-hand notation for the integrals in order to keep the notation manageable. Our short-hand notation for the integrals appears in column 3. In column 5 we comment on their reality property and tick off those integrals that have been completed up to now. As mentioned before all one-loop integrals have been done except for the four-point integral D_1 involving three massive propagators. We have compared our results to results in the literature whenever possible and when these were accessible. We have found agreement. In column 4, finally, we indicate which of our results are new.

We mention that one also needs the imaginary parts of the amplitudes since the square of the amplitude contains also imaginary parts according to

$$|A|^2 = (\text{Re}A)^2 + (\text{Im}A)^2. \quad (1)$$

Note, though, that the imaginary parts are only needed up to $\mathcal{O}(\varepsilon^1)$ since the IR/M singularities in the imaginary parts of the one-loop contributions are of $\mathcal{O}(\varepsilon^{-1})$ only.

The scalar four-point integrals are the most difficult to calculate. They contain a very rich structure in terms of polylogarithmic functions. For example, the ε^2 -coefficients of the Laurent

series expansion of the four-point integrals contain logarithms and classical polylogarithms up to order four (*i.e.* Li_4) (in conjunction with the ζ -functions $\zeta(2, 3, 4)$) and products thereof, and a new class of functions which are now termed multiple polylogarithms [7]. A multiple polylogarithm is represented by

$$Li_{m_k, \dots, m_1}(x_k, \dots, x_1) = \int_0^{x_1 x_2 \dots x_k} \left(\frac{dt}{t} \circ \right)^{m_1-1} \frac{dt}{x_2 x_3 \dots x_k - t} \circ \left(\frac{dt}{t} \circ \right)^{m_2-1} \frac{dt}{x_3 \dots x_k - t} \circ \dots \circ \left(\frac{dt}{t} \circ \right)^{m_k-1} \frac{dt}{1-t},$$

where the iterated integrals are defined by

$$\int_0^\lambda \frac{dt}{a_n - t} \circ \dots \circ \frac{dt}{a_1 - t} = \int_0^\lambda \frac{dt_n}{a_n - t_n} \int_0^{t_n} \frac{dt_{n-1}}{a_{n-1} - t_{n-1}} \times \dots \times \int_0^{t_2} \frac{dt}{a_1 - t_1}.$$

Instead of using the multiple polylogarithms of Goncharov we have chosen to write our results in terms of one-dimensional integral representations given by the integrals

$$F_{\sigma_1 \sigma_2 \sigma_3}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) =$$

$$\int_0^1 dy \frac{\ln(\alpha_1 + \sigma_1 y) \ln(\alpha_2 + \sigma_2 y) \ln(\alpha_3 + \sigma_3 y)}{\alpha_4 + y}$$

and

$$F_{\sigma_1}(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = \int_0^1 dy \frac{\ln(\alpha_1 + \sigma_1 y) \text{Li}_2(\alpha_2 + \alpha_3 y)}{\alpha_4 + y},$$

where the σ_i take values ± 1 and the α_j 's are combinations of the kinematical variables of the process. The numerical evaluation of these one-dimensional integral representations are quite stable.

A comment on the length of our expressions is appropriate. The untreated computer output of the integrations is generally quite lengthy. The hard work is to simplify these expressions. We have written semi-automatic computer codes that achieve the simplifications using known identities among polylogarithms. All intermediate steps have been checked numerically. Even after simplification, the final expressions are generally too long to be given in this presentation. As mentioned earlier on, we have compared our results to the results of other authors when they were available and have found agreement.

4. Laurent series expansion of one-loop amplitudes

In order to obtain the Laurent series expansion of the full one-loop amplitude one has to combine the Laurent series expansion of the scalar one-loop integrals with the ε -expansion from the spin algebra calculation and the Passarino-Veltman decomposition. This is the subject of a second paper which we are working on [8]. As an example of such an amplitude result we present results on the contribution of the one-loop gluon triangle graph to the process $g + g \rightarrow Q + \bar{Q}$. We shall only list the ε - and ε^2 -terms for the triangle graph contribution since the finite and divergent pieces have been calculated before (see e.g. [9]).

Let us begin by defining a reduced amplitude which is obtained by extracting the spin wave functions from the full amplitude M , *i.e.*

$$M = \varepsilon(p_1) \varepsilon(p_2) \bar{u}(p_3) M^{\mu\nu} u(p_4). \quad (2)$$

For the reduced gluon triangle amplitude $M_{(\text{tri})}^{\mu\nu}(g)$ with gluons and ghosts inside the triangle loop one obtains

$$\begin{aligned} M_{(\text{tri})}^{\mu\nu}(g) = N_C \{ & -3\varepsilon(B_s^{\mu\nu}[207B_5^{(1)} + 12B_5^{(0)} \\ & + 54C_4^{(1)}s + 8] + 6i(T^a T^b - T^b T^a) \not{p}_1 (g^{\mu\nu} \\ & [9B_5^{(1)} - 12B_5^{(0)} + 9C_4^{(1)}s - 8]/s + 8p_2^\mu p_1^\nu [3B_5^{(0)} \\ & + 2]/s^2)) - \varepsilon^2(B_s^{\mu\nu}[621B_5^{(2)} + 36B_5^{(1)} + 24B_5^{(0)} \\ & + 162C_4^{(2)}s + 16] + 6i(T^a T^b - T^b T^a) \not{p}_1 (g^{\mu\nu} \\ & [27B_5^{(2)} - 36B_5^{(1)} - 24B_5^{(0)} + 27C_4^{(2)}s - 16]/s \\ & + 8p_2^\mu p_1^\nu [9B_5^{(1)} + 6B_5^{(0)} + 4]/s^2)) \} / 324. \quad (3) \end{aligned}$$

The one-loop triangle graph amplitude $M_{(\text{tri})}^{\mu\nu}(g)$ has been written in terms of the Laurent series expansion of the two massless scalar two-point and three-point one-loop integrals B_5 and C_5 listed in Table 1, where the Laurent series expansions are defined by

$$B_5 = iC_\varepsilon(m^2) \left(\sum_{n=-1}^N \varepsilon^n B_5^{(n)} \right), \quad (4)$$

and

$$C_4 = iC_\varepsilon(m^2) \left(\sum_{n=-2}^N \varepsilon^n C_4^{(n)} \right), \quad (5)$$

where $N = 2$ in our application. The Born term amplitude $B_s^{\mu\nu}$ appearing in (3) is defined by [9]

$$B_s^{\mu\nu} = 2i(T^a T^b - T^b T^a)(g^{\mu\nu} \not{p}_1 + p_2^\mu \gamma^\nu - p_1^\nu \gamma^\mu)/s, \quad (6)$$

and the coefficient $C_\varepsilon(m^2)$ is given by

$$C_\varepsilon(m^2) = \frac{\Gamma(1+\varepsilon)}{(4\pi)^2} \left(\frac{4\pi\mu^2}{m^2} \right)^2. \quad (7)$$

After insertion of the appropriate coefficient functions $B_5^{(n)}$ and $C_4^{(n)}$ our results can be seen to fully agree with the results of the authors of [10] who also calculated the gluonic one-loop corrections to the three-gluon vertex with one off-shell gluon.

Eq.(3) gives an impression of the interplay of the three different sources of positive ε -powers in the amplitude calculation mentioned earlier on. Note in particular that different orders of the Laurent series coefficients of the scalar integrals enter

at each order of the Laurent series expansion of the full amplitude.

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REFERENCES

1. P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. **B303** (1988) 607; *ibid* **B327** (1989) 49; *ibid* **B335** (1990) 260(E).
2. W. Beenakker, H. Kuijf, W. L. van Neerven and J. Smith, Phys. Rev. D **40** (1989) 54 ; W. Beenakker, W. L. van Neerven, R. Meng, G.A. Schuler and J. Smith, Nucl. Phys. **B351** (1991) 507.
3. M. Glück, J.F. Owens and E. Reya, Phys. Rev. D **17** (1978) 2324; B. L. Combridge, Nucl. Phys. **B151** (1979) 429; J. Babcock, D. Sivers and S. Wolfram, Phys. Rev. D **18** (1978) 162 ; K. Hagiwara and T. Yoshino, Phys. Lett. **80B**, (1979) 282; L. M. Jones and H. Wyld, Phys. Rev. D **17** (1978) 782; H. Georgi *et al.*, Ann. Phys. (N.Y.) **114** (1978) 273 .
4. M. Cacciari, S. Frixione, M. L. Mangano, P. Nason, G. Ridolfi, ArXiv: hep-ph/0312132.
5. J.G. Körner, Z. Merebashvili and M. Rogal, “Laurent series expansion of massive scalar one-loop integrals to $\mathcal{O}(\varepsilon^2)$ ”, to be published.
6. K. G. Chetyrkin and F. V. Tkachov Nucl. Phys. **B192** (1981) 159 ; F. V. Tkachov, Phys. Lett. **100B** (1981) 65.
7. A.B. Goncharov, Math. Res. Lett. **5** (1998), available at <http://www.math.uiuc.edu/K-theory/0297> .
8. J.G. Körner, Z. Merebashvili and M. Rogal, “One-loop amplitudes for $g + g \rightarrow Q + \bar{Q}$ and $q + q \rightarrow Q + \bar{Q}$ to $\mathcal{O}(\varepsilon^2)$ ”, to be published.
9. J.G. Körner, Z. Merebashvili, Phys. Rev. D **66** (2002) 054023.
10. A.I. Davydychev, P. Osland and O.V. Tarasov, Phys. Rev. D **54** (1996) 4087; *ibid* (1999)109901(E)